

Motion Math: Perceiving fractions through embodied, mobile learning

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Abstract

Elementary school students often struggle with fractions, inhibiting progression to pre-algebra and other advanced topics; even students who show proficiency with fractions lack a deeper sense of how multiple representations of fractions (pie charts, percents, number lines, etc) relate to one another. In addition, research suggests that estimation skills correlate strongly with math achievement (Schneider, 2006, Johanning, 2006). The technology of accelerometers, embedded in iPhones and other mobile devices, affords new experiences at the nexus of games and curriculum that improve students' "gut sense" of fractions by encouraging embodied cognition. Our prototype game, Motion Math, is a motivating learning game that demonstrates the unique affordances of mobile technology, and helps students perceive fractions in multiple forms.

Learning problem

Albert Einstein once remarked to French mathematician Jacques Hadamard, "The words of the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought...are, in my case, of visual and some of a muscular type" (Hadamard 1945, pp. 142-3, cited in Root-Bernstein & Root-Bernstein, 2008). By imagining himself as a photon moving at light speed, Einstein placed his body inside the problem space and made a breakthrough discovery—relativity. What innovative discoveries await students who—from an early age—acquire not just a symbolic or visual, but an intuitive, *muscular* understanding of mathematics?

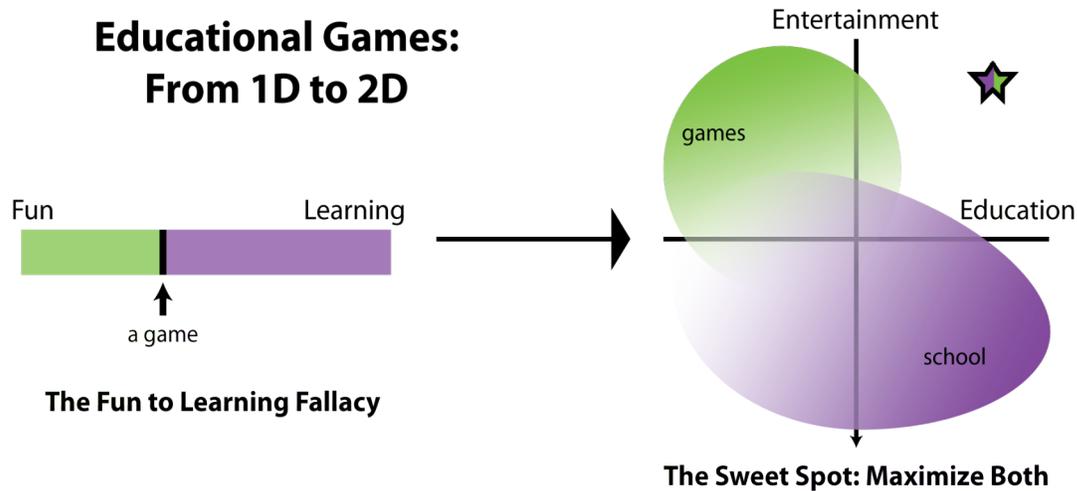
Motion Math was first conceived as a result of practical experience that showed the need for this type of intuitive understanding. Carlos, a fifth grade student we tutored, worked on fraction addition problems, and his lack of understanding became apparent. After completing several steps of adding dissimilar fractions, Carlos made errors because he likely had no "gut sense" of how big $\frac{2}{3}$ plus $\frac{1}{4}$ should be. The fraction existed for him as a purely numeric concept: a 2 placed over a 3. Motion Math was created as a platform to quickly give learners like Carlos intuitive visual and physical judgments of numerical value. Research supports the choice of fractions as an important problem area:

"Teachers and researchers have typically described fractions learning as a challenging area of the mathematics curriculum (e.g., Gould, Outhred, & Mitchelmore, 2006; Hiebert 1988; NAEP, 2005). The understanding of part/whole relationships, procedural complexity, and challenging notation, have all been connected to why fractions are considered an area of such difficulty. Teachers and researchers have struggled to find ways to make fractions more meaningful, relevant and understandable to students." (Bruce and Ross, 2009).

Within the diverse realm of fractions education, we've chosen the number line as an important area because research affirms "the importance of external number line estimation competencies for children's mathematical school achievement in addition to conceptual understanding and intelligence" (Schneider, 2006. Also see Johanning, 2006, p2). Indeed, number line competence may play a key role in mathematics cognition; evidence from mental modeling research suggests that "approximate mental arithmetic involves dynamic shifts on a spatially organized mental representation of numbers" (Knops & Viarouge, 2009).

With this research and our personal experience in mind, our project addresses three learner needs: embodied learning, motivating experiences, and differentiated instruction. Most centrally, we aim to address the potential of embodied experience to deepen content mastery. Embodied cognition theory states that even abstract concepts are based in our perceptual systems and the interaction of our bodies with the environment (Lakoff and Núñez, 2000). In terms of our project, this implies that the abstraction of math can be understood more deeply if it is tied to the physical interaction of the player. We seek to verify this connection. A potential misconception is that standard classroom work is "un-embodied"; however, everything humans do to control an interface is embodied. Moving a pencil on a page and clicking a mouse—even moving ones eyes to control an eye-tracker interface—are actions involving the body. Our project is based on the premise that a novel gesture that maps more closely to the concept of a line in space, will lead to better understanding.

Secondly, our project addresses the need of learners to have a fun, motivating experience that appropriately uses games to promote learning. Integrating curriculum and game design processes is challenging work. Too often we hear people characterize an educational game as lackluster and justify its mediocrity by saying: "Well, it's for education." The difficulty of designing quality games with educational purposes has led to the belief that one must sacrifice education for entertainment or vice versa. We seek to re-imagine the space of educational games by developing processes for incorporating endogenous learning experiences into games (see graph below). Our efforts have been greatly informed by a close reading of Malone and Lepper's 1987 article 'Making Learning Fun: A Taxonomy of Intrinsic Motivations for Learning'. This process will lead to a 'sweet spot' of experience that will maximize learning and entertainment values.



Lastly, our game’s problem generator addresses the need to have differentiated instruction appropriate for each individual learner. A major problem of student motivation is differentiation of abilities; strong students tend to be bored with class exercises while weak students get progressively more behind (Stipek, 2002). Motion Math addresses this issue with a generator that personalizes the problem sequence for each player based on performance; a player who is struggling with thirds, for example, will see more thirds problems until they have shown mastery.

Existing solutions

An example of an embodied interaction interface that reinforces math skills is Perfect 10, an element of the Wii Fit Plus game suite. In the mini-game, players must select numbers that add up to ten by leaning in different directions on the Wii Balance Board. Though math is endogenously integrated into the gameplay, the scope of the game is extremely limited (there’s a small set of between two and four numbers that add up to ten).

An example of a math curriculum that stresses embodied cognition is the Rational Number Project, which “placed particular emphasis on the use of multiple physical models and translations within and between modes of representation--pictorial, manipulative, verbal, real-world, and symbolic” (Cramer, et al., 2002) and has improved post-test and retention test scores. Computer-based math programs such as Cognitive Tutors and Intelligent Tutoring Systems (Melis & Siekmann, 2004) have proven effective at increasing student mastery and reducing the time needed to reach mastery; these programs inspire our use of a differentiated problem generator. We find a similar principle in the “active dynamic difficulty adjustment design philosophy” of game innovator Jenova Chen (*Flower, Cloud, Flow*) and the work of Kamran Sedig (2007) defining an algorithm for the right level of challenge in

learning games. Research from these diverse fields – math education, embodied cognition, and game design – will continue to inform our development decisions.

Design approach

Three types of playability heuristics for mobile games defined by Korhonen and Koivisto (2007) informed our design: usability, mobility, and gameplay. The usability category inspired features like implementing convenient, flexible controls and providing immediate, diverse feedback. We addressed mobility with allowances for quickly starting and stopping; the game can be played anywhere, for a minute or an hour, with almost all of the time devoted to on-task learning. Finally, our gameplay features include player challenges and motivations, clear goals, and consistent indications of progress. The rhythmic movement and increasing challenge keep players engaged, reducing the "potential for students to become distracted, diminishing educational engagement" (Shuler, 2009, p25).

Motion Math's gameplay is particularly generative for the field of mobile learning, given that our game mechanics, scaffolding, and assessment all occur without any textual input; the minimalist game interface counters "the disadvantages and limiting physical attributes of mobile devices" (Shuler, 2009, p34).

Our game design is based on a foundational text by researchers Malone and Lepper. They stipulated that endogenous integration of educational content into games enhanced learner motivation. In a well-developed educational game experience, "some integral relationship [exists] between the instructional content and motivational embellishments" (Malone and Lepper, 1987, p 232). In Motion Math, we have created a game mechanic whereby content knowledge (in this case fractions) is essential to progressing in the game (by maneuvering the ball to the appropriate location). As we consider integrating new content areas and novel gestures into the game, we will return to Malone and Lepper's taxonomy to verify that the learning remains intrinsic to the game mechanics.

Our iterative user testing with dozens of students in homes and schools led to many changes in our game's design. One example of a design feature that's been very informed by user testing is how the game begins. At first, when we put the paper prototype in front of our first tester, we realized that our gameplay mechanic of tilt was unclear. In our first digital prototype, we put a large ball containing text instructions: "Move the fraction-ball to the right place on the number line by *tilting*." Users would either not read these instructions or read them and still be unclear on how to play. Our next iteration was to put math content instructions in the first level that said, "Your first problem is $\frac{1}{2}$ ". This helped some players, but others were still unclear as to how to maneuver the ball. Our most recent evolution is to introduce the players first to the tilt mechanic by having them maneuver a colored ball (which doesn't contain any math content) to a similarly colored block. This mini-training level successfully introduces the physical game mechanic, and allows us, in the first level of the game, to bring in math content. Continued user testing has led to even more design insights.

Technology

Motion Math can only exist on a mobile platform. Mobile affordances such as the accelerometer, the hand-held interface, and inherent mobility are essential components of the game. Increasingly embedded into mobile devices, accelerometers offer a unique, intuitive, underutilized interaction style. In addition to the primary game mechanic of tilting, players can dunk the ball by snapping the device forward and also tip the device to access secret levels. While the Nintendo Wii was the first platform to popularize gestural input via accelerometers (Shuler, 2009, p30), mobile devices offer an even more satisfying physical interaction: both the controller and the game environment fit in the player's hands, directly connecting the learner to content.

Many new technologies for education have been criticized because students waste time becoming proficient with or exploring the tool rather than engaging the learning content. Motion Math has a straightforward, intuitive interface that both students and teachers have used with no prior experience or documentation, integrating the unique affordances of the mobile platform.

Solution

Currently, we have a working iPhone game that uses a bouncing ball metaphor to teach students fraction estimation. Students control the movement of a bouncing basketball containing a fraction by tilting their iPhone; their goal is to land the fraction-ball near its correct place on a number line that goes from 0 to 1. Once they aim and land the fraction-ball at approximately the right position, a new fraction appears on the bouncing ball and the student must tilt it to a new location. Scaffolded hints help students with challenging material. Higher levels expand the variety of fractions, increasing denominators up to 20ths, and at level five, introduce the concept of improper fractions. The entire construction of the game is intended to keep students in a state of continuous, engaged flow; every moment of the game is on-task learning. We chose our metaphor because a bouncing ball functions as its own timer; the fast pace keeps the game fun and reinforces the goal of estimation - there's no time to wait and worry about aiming too precisely.

We are further improving the game by redeveloping it on a more flexible, solid platform that will allow us to easily experiment with diverse forms of fractions, and measure the game's impact on content mastery and motivation. In addition, we plan to continue to improve the game's graphics, sound effects, and gameplay.

Assessment

We ran a pilot test with a pre-test post-test control group design on 12 summer camp students in grades 2 – 7. The pre- and post- test consisted of 32 questions relating to fractions, drawn from California State Standards for math for grades 3 and 4. We randomly

switched similar questions from the pre- and post-test to attempt to make them roughly equivalent in difficulty. Students randomly selected from the control group experienced a 10-minute outdoor classroom activity in which they placed fractions on an 80-foot-long rope (an embodied experience of the content.) Students in the experimental group spent 10 minutes playing our game.

The results of the study were inconclusive; we were not able to show a statistically significant difference between any of the tests or groups. We learned that:

- Although our game appeals to children of many ages, for the purposes of a meaningful pre- and post-test we need a more focused age range of student.
- We need a larger sample size and/or a longer period of time for students to engage in the activity.
- Lastly, perhaps more importantly, we realized that our game is best thought of not as a substitute for all fractions knowledge and fractions skills, but rather one specific, fundamental skill: the perception of fractions in multiple forms. Future experiments will test the efficacy of our product in this dimension.

As a user-test however, the experiment was quite successful. Students' and parents' enthusiasm for the game, as well as their design suggestions, were encouraging. We are motivated to continue to explore how technology can aid students' understanding of fractions and other difficult mathematical subjects.

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Amrita Thakur - Initial design and engineering collaborator

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Rachael Shepherd - Teaching collaborator

Professor Erica Robles - Experiment consultant

Professor Clifford Nass – Personalization IP collaborator

Dave Baggeroer – Design consultant.

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EDUCATION

Stanford University, School of Education, Stanford, CA Expected 2010

MA in Education: Learning, Design, and Technology

Relevant coursework: Technologies for Learners, Understanding Learning Environments, Economics of Education, Design of Technologies for Casual Learning, Curriculum Construction, iPhone and Flash Development

Stanford University, Stanford, CA 2006

BS in Computer Science

Relevant coursework: Human-Computer Interaction Design, Computer Graphics and Imaging, Designing Characters for Computer Games, Artificial Intelligence, Programming Paradigms

EDUCATION TECHNOLOGY EXPERIENCE

Castilleja School, Palo Alto, CA

Educational Consultant

03/09-08/09

- Re-imagined underutilized learning spaces around the school in preparation for a 1:1 student:laptop program.
- Proposed a large library redesign to a board member along with heads of IT, Grounds, the Library, and School.
- Leveraged numerous visualizations developed through conversations with the head librarian.
- Researched current practices of learning spaces and 1:1 laptop integration in publications and through conversations with students and internal and external educators.
- Developed an internal website that collated all my research and creations.

Umeå Universitet, HUMlab, Umeå, Sweden

Curriculum Designer and Co-Instructor, [Game Gurus](#)

03/09-08/09

- Designed two-session curricula to teach 8 to 9-year-olds how to program games and interactive stories in [Scratch](#), a game prototyping application for young developers.
- Created online games for multimedia tutorials to teach different game mechanics used in students' prototypes.
- Generated marketing and teaching materials for future instructors.
- Utilized HUMlab's Screenscape (a space of 15 giant, HD displays) for a final presentation of children's projects to parents and local media.
- Interviewed by the press and featured on local and national TV and radio broadcasts.

Workshop Leader & Co-Organizer, [Programming for Humanists](#)

03/09 – 04/09

- Conducted dialogues between professional computer scientists and literature and humanities scholars.
- Taught basic programming skills and advanced processes and practices.
- Discussed analogues between programming and literary analysis.
- Reached goal of getting literature-oriented scholars comfortable with programming languages.

Artist and Interaction Designer, [Miss Baltazar's Laboratory](#)

02/09-07/09

- Create interactive art exhibit with Carl-Eric Enqvist ([Visionary Existence Laboratory](#) founder) and Stephanie Wuschitz (Miss Baltazar's Laboratory founder) for a contemporary art museum in Umeå, Sweden.
- Leveraged [Processing](#) and [Arduino](#) as technologies for interactive aspects of the final product.
- Guided users to a giant interactive starfish that sang music notes which users mimicked by pressing buttons.
- Participated as a panelist with Carl-Eric and Stefanie on a public panel to discuss our creative processes.

Keynote Speaker, Digital Interaction in the North

04/22/09

- Keynote speaker at gathering of CEOs and other professionals in software, multimedia, and film industries.
- Focused on improving projects through agile development processes motivated by up-and-coming developers.

Galileo Learning, Oakland, CA

Curriculum Design Consultant, [Galileo Summer Quest](#)

11/08 – 02/09

- Designed Video Game Design course emphasizing leadership, logic, problem solving, and collaboration skills.
- Focused on making the curriculum accessible to teachers with non-technical background.

Lead Video Game Design Instructor, [Galileo Summer Quest](#)

04/08 – 08/08

- Founding member of this Galileo program for 5th – 8th graders, helped conceptualize the program.

JACOB KLEIN

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EDUCATION

- Stanford University, School of Education**, Stanford, CA Expected 8/10
MA, Education in Learning, Design, and Technology. Course highlights: Web-based Technologies in Education, Design Thinking Bootcamp, Casual Learning on the iPhone, Social Software.
- Stanford University**, Stanford, CA 5/02
BS, Symbolic Systems, GPA: 3.71
Course highlights: Foundations of Cognition, HCI, C++, Theory of Knowledge, AI, Improvisation.
- New York University**, NYC 8/00
Intro to Digital Video Production

VIDEO PRODUCTION EXPERIENCE

- The NewsHour with Jim Lehrer**, New York, NY; Beijing, China 4/05–9/05
Freelance Producer
- Co-produced and edited *China on the Rise*, PBS series on the Chinese economy, from initial story concepts and storyboards through final video deliverables.
 - Awarded the Loeb Award, business journalism's top honor.
- Seed Media Group**, New York, NY 4/06–2/08
Company Video Producer
- Produced, shot, and edited videos exploring intersection of science and society.
 - Filmed conversations with Michel Gondry, David Byrne, Jonathan Lethem, E.O. Wilson.
 - Reported on 2006 International AIDS Conference featuring Bill Clinton and Bill Gates.
 - Prototyped promotional video concepts with storyboards and user testing.
- SlateV.com**, New York, NY 6/07–12/07
Freelance Producer
- Produced, shot, and edited videos on topics including Presidential politics, piracy, accents.
- Intel International Science and Engineering Fair**, Albuquerque, NM 3/07–5/07
- Managed ten-person crew to produce video series for Intel website.

TEACHING EXPERIENCE

- KIPP Summit Academy**, San Lorenzo, CA 1/09–6/09
7th Grade English Teacher
- Created and taught project-based curriculum incorporating literature, writing, grammar.
 - Raised class score on year-end STAR test by 15%.
- Freelance tutor**, New York, NY 9/02–6/06
- Tutored elementary, middle school, and high school students in Math, Writing, English.
 - Developed tests, writing assignments, tutorials, and test-taking strategies for students taking the SAT I, SAT II Math and Literature, SSNSAT, and Stuyvesant entrance exams.

CAMPAIGN EXPERIENCE

- Obama for America**, Columbus, OH; Phil., PA; Atlanta, GA; Miami, FL 1/08–11/08
Regional Field Director, Field Organizer, New Media Volunteer
- Managed and trained 13 field organizers covering suburban Atlanta region w/ 1 mil. residents.
 - Led two-day training on persuasion, empowerment, and viral growth for 200 volunteers.
 - Modeled diverse voter profiles to fine-tune persuasion efforts.
 - Coordinated community and political efforts with regional voter contact operations.
 - Produced video portraits of volunteers for campaign website.

TECHNICAL SKILLS

Final Cut Pro 7, Panasonic HVX200 video camera, Javascript, C, C++, HTML, CSS, digital photography, Photoshop CS3