The Middle School Mathematics through Applications Program:
A Case in Learning Design

Case Problem # 2 – Proposed Redesign
for
ED 333A

Submitted to Dr. James Greeno and Dr. Deedee Perez-Granados

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I. Background Information

Middle school math classrooms have traditionally focused on abstract problem solving skills and drilling students in those abstract skills. Research has shown that students develop a deeper understanding of concepts through practical application. The research and design team who created Middle School Math through Applications Program (MMAP) specifically designed math curricula which demonstrate practical applications as they relate to math concepts.

The program was designed to meet middle school math standards. The Antarctica Project is a unit within MMAP, which students are taken through the process of building a workstation in Antarctica. It is a learning environment where students use math concepts to solve building problems. The students use a computer application called ArchiTech, similar to a CAD application to construct their workstations. The entire class works concurrently on the same assignment. The students work in groups of three sharing one computer.

The curriculum intended to teach students practical applications of mathematics concepts, but the model brought with it a new set of problems. The designers felt a loosely-constructed curriculum around the task of creating an Antarctic workstation would leave room for teachers to maintain their own teaching styles throughout the unit. Some teachers were able to effectively use this flexible curriculum to their advantage; however it was seen that many were not. Assessments showed that many teachers were missing opportunities to capitalize on students’ understandings or misunderstandings of specific math concepts in order to teach related and tangential math concepts.

Another problem which arose from this new curriculum was that the teachers were noticing that students were not making strong connections between math concepts and the building of the workstation. Instead of connecting the Antarctica Project to learning how to apply concepts to situations, they thought it was just fun to be creative, and they did not understand that it was indeed math they were learning. The teachers received phone calls from concerned parents that the students described what they did in math class that day as “played with computers” and “being an architect.” It was clear that students were not making the right connections.

It was also seen that not all students understood the concepts equally, a concern of both the teachers and the designers. It was also seen that groups of students were progressing at different rates. The teachers had a difficult time assessing each student’s understanding of the math being presented. It was difficult for the teacher to be in 10 places at once in a class of 30. The teacher’s ability to make sure everyone was learning what they needed to be learning, when they need to be learning it, was failing.
II. The Learning Problem

The conditions described above have led us to the following conclusion:

We believe the current MMAP curriculum and teacher’s guide is not providing enough support for teachers in three ways:

1. Learning to better identify teachable moments beyond what is laid out in the current curriculum
2. Helping students draw connections between math concepts and real-life applications
3. Assessing the student’s learning

III. The Learning Goals

Our goals in redesigning the MMAP model are the following:

1. Help teachers develop ability to rely more on strategic knowledge. This includes creating a learning environment for teachers in which they can further develop their strategic knowledge skills. The teachers need to learn more how to apply just the right piece of knowledge at just the right time. (Gott, 1988-1989)

   This learning environment will:
   a. Help teachers identify teachable “moments”
   b. Help teachers apply math concepts in that moment
   c. Help teachers facilitate knowledge equally across all groups
   d. Improve teachers’ ability to understand student’s learning

2. Help students draw connections between math concepts and real-life applications

3. Help teachers assess their students’ learning

IV. The Design Principles

Inherent in the current curriculum is reliance on the teacher’s strategic knowledge to seize moments throughout the unit to take the student’s learning one step further.

Our solution draws upon both cognitive and situative design principles as identified by Greeno, et al. We mainly focus on the cognitive model of learning as identified by Greeno et al and Gott. The following assumptions are the foundation of our redesign:

1. Teachers are well-versed in teaching procedural math knowledge (teaching students to solve a math equation)
2. Teachers are lacking strategic knowledge to facilitate new levels of understanding within the students (teaching students how to apply math concepts to situations)
We believe an effective way to help teachers develop strategic knowledge is through the coordination of teaching procedural knowledge and the application of that knowledge to a real-life situation.

Create a learning environment for teachers in which they can further develop their strategic knowledge skills. (Teacher Orientation)
The designers purposely left the curriculum flexible to accommodate teaching styles of individual teachers. What the designers didn’t account for was that teachers were lacking strategic knowledge which would enable them to plan for identifying teachable moments and to then choose a course of action with that teachable moment. They need strategic knowledge for planning, evaluating and choosing courses of action (Gott, p101), in order communicate how to understand math from a real-life situation.

Our solution proposes a teacher’s orientation to aid in building this skill. A blend of a cognitive and situative learning environment will serve to foster strategic knowledge development in the teachers and build greater understanding through teamwork. Through this workshop exercise, the teachers will:

• Begin to associate teachable moments with their own experience in the teacher training workshop with their student’s experience in their classroom. The skills learned and the identification of teachable moments in this case are more likely to be retained than if they are simply taught the skills, or given a chart to help them identify the key points. “A repeated finding is that general strategies directly taught to students tend not to be performed spontaneously used under conditions different from those in which they were initially practiced.” (Greeno et al, p35) (e.g. A.L. Brown and Campione, 1977)

• Engage in meaningful tasks around their own curriculum. Customize and modify their curriculum to reflect their own understanding as well as teaching style. This will help scaffold their process of becoming more adept at identifying the teachable moments. They will learn to construct mental models of their own curriculum, which will enable them to better teach the units to the students.

• Develop their own deeper understanding of the material, creating a more positive epistemological identity which will allow them more confidence in teaching beyond the established curriculum.

Help teachers apply math concepts in that moment (Teacher Orientation)
Identification of the teachable moment needs to lead to actual mathematics instruction. The workshop we have designed will reflect this need, through bolstering teachers’ metacognitive and strategic thinking abilities in knowing which concept to apply when.

Help teachers facilitate knowledge equally across all groups (Curriculum Redesign)
A change in the curriculum will involve the small student groups working together, supporting other groups and holding peer discussions about the material being covered. Situative learning theory suggests that students who are actively participating in discussion of the math concepts presented within an activity are more likely to become attuned to the overall mathematics domain. Research by Barron has also shown that success or failure of student group work depends highly upon social interrelation factors. The student groups will rotate members throughout the activities in order to promote more equally distributed knowledge.
Help students draw connections between math concepts and real-life applications (Curriculum Redesign)
According to Bruner, instruction should be “spiral” in structure so the student can build gradually upon what they have already learned. The new curriculum will be structured so that as the student is learning basic building concepts, they will also be learning stage-appropriate math concepts.

Another aspect of the curriculum redesign is that the students will be told in advance the concepts they will be learning as they relate to the activity. Once told what they will be doing, students will be more likely to understand what they are learning while they are learning it, than if the teacher had not established the context.

Improve teachers’ ability to understand student’s learning (Assessment)
We have designed an assessment rubric for the teacher to gauge the students’ ongoing work. This will help the teacher assess the process of the students reaching a solution, as well as the depth of the student understanding.

V. Teacher Orientation
The goal of our teacher orientation is to provide conceptual framework for planning and identifying opportunities for instruction within the context of Antarctica Project activities. The workshop aims to foster a greater familiarity with the activities that students engage in, and to develop connections between specific mathematical principles and these activities.

Ultimately, teachers should be proficient in recognizing “teachable moments” as well as intuitively use these moments to enhance the learning of his/her students. This orientation will provide teachers with multiple models for visualizing and identifying mathematical concepts within realistic activities.

The orientation leverages the shared knowledge of teachers through collaborative exercise, as well as personal knowledge through metacognitive exercise. Learning exercises are also designed around the solution of problems that teachers will encounter naturally in the classroom as well as while developing curricula throughout the school year.

Orientation details

- Summer Workshop
- Three days long
- Four hours per day

Orientation Introduction
The orientation kick-off entails introductions of both orientation leaders and new trainees. Orientation leaders introduce themselves as volunteers who have successfully used the new curriculum and have been involved in its implementation throughout the year. They explain the challenges and benefits of the new system as well as present to the group the learning objectives of the workshop. They inform trainees of the activities they will be engaging in, as well as familiarize them with the basic concepts that will be covered during instruction.
Individually, teachers are asked to identify and list the mathematical concepts that he/she must cover during the school year. Orientation leaders will identify trainees with similar goals and this list is kept for future reference. Teachers are finally asked to break up into groups of three to begin first set of activities.

**Activity 1: Teachers Engage in Antarctica Project Activities**

1. **Project Activities**

   In groups of three, the complete class engages in and completes a series of building exercises in the Antarctica Project. During each activity collaborating teachers take clear notes on their problem-solving processes including mathematical notations, equations, thought processes. Teachers also take notes on the specific activities they engaged in and the math concepts involved in solving those particular problems.

2. **Group Discussion**

   Teachers return to the larger group to share their solutions and processes with other group members. Orientation leaders facilitate a discussion of the challenges of the activities presented by ArchiTech and challenge the group to consider the many types of problem-solving that was required for each. Supported by facilitators, teachers complete a list of the activities and the math concepts encountered through each exercise. Teachers are provided an empty Math Opportunities Chart. As a group, teachers complete the chart, filling out the math concepts associated with particular activities.

**Activity 2: Antarctica Project Observation**

1. **Observing for “teachable moments”**

   Teachers use the Math Opportunities Chart constructed in Activity 1 as a framework for perceiving mathematical concepts within the context of activities. The class is asked to divide into two equal parts to engage in the following exercises. After completing one round, the groups switch roles and complete the activities of the other.

   - **Group 1**- These teachers return to complete more Antarctica Project activities. Activities are broken into 7 shorter components, constructed around the same type of math challenges (algebra, fractions, measures, etc.) These “Activity Blocks” are meant to isolate activities based upon 1-3 conceptual challenges at a time. During this exercise, teachers do not take notes beyond what is necessary to complete mathematical equations. At the completion of each activity block, a new activity is undertaken.

   - **Group 2**- These teachers observe the exercises and practices of Group 1. They take notes on the math challenges encountered by Group 1 during each Activity Block. After each block, orientation leaders and both groups discuss which activities required which types of mathematical problem solving. Further discussion is encouraged on the possibilities of more relevant and tangential math concepts that might serve to further illustrate concepts and extend conceptual understandings for students. Orientation leaders contribute their past experiences in identifying these same math opportunities. The group provides peer assessment and support of individual’s abilities in recognizing math opportunities within each activity.
Activity 3: Teachers Design a Curriculum

Teachers use original list of math requirements generated at the onset of class with the Math Opportunities Chart created in Activity 1. This process is facilitated by orientation leaders identified with similar teaching goals. Teachers are scaffolded in the use of the chart in mapping the curricular requirements of their classes to specific activities presented by the Antarctica Project. Mathematical concepts in the chart are ordered in a sequence in which simpler concepts are followed by related, and more complex concepts. The chart is constructed by matching required math concepts to the pre-determined activity list ensuring that math concepts will be presented in an appropriate sequence, based on logical conceptual development.

This artifact of the course is a curricular framework for future activity-based instruction based upon the teaching objectives of each instructor. This self-created working model of the Math Opportunities Chart can serve as a reference for developing future curriculum, as well as a “hit list” for required math concepts. It will be used as a tool for instruction in the classroom, as well as a point of discussion outside of class throughout the year.

Orientation Follow-up

At the end of the workshop, teachers are asked to share their own Math Opportunity Charts. Teachers compare and exchange ideas and incorporate elements that they find relevant in the findings of their peers. During this exercise teachers are asked to form small “buddy” groups with teachers that share their same teaching goals. With their buddy, teachers are asked to work together during the school year to support one another. This support entails the sharing of challenges met in class as a result of the new instruction, collaboration on solutions to those challenges and the development of newer versions of the curriculum.

Buddy groups are also asked to meet with orientation leaders as well as with the larger groups twice more throughout the school year to share their understandings with other faculty and to discuss adaptive strategies. Teachers provide feedback on which activities proved effective in teaching concepts and which activities lead to the discovery of new opportunities to teach. This can be accomplished with face-to-face meetings, or via an online discussion board available to all project participants. Groups also share their assessment of their own success in harnessing the opportunities of their own curricula and whether they felt a stronger sense of empowerment. In the final session, buddy groups are asked to be volunteer instructors in the following year’s orientation.

VI. Redesigned Curriculum

Our curricular development process was dually focused. First, we intended to provide support for teachers involved in the MMAP program. Second, we planned to provide resources for teachers in assessing their individual student’s or group’s readiness for the required mathematical skills or concepts. We agreed with, and intended to keep, the R & D team’s original goal – to provide an environment in which mathematical concepts are surrounded by authentic tasks.

Based on our concerns with the original design of the curriculum, our goals here are to (1) help teachers discover and capitalize on math opportunities, (2) help students, individually and as a
group, make connections between their project activities and mathematical concepts, and (3) support teachers in assessing their students’ learning.

Overall, the redesigned curriculum is more structured in helping teachers take advantage of math opportunities. Along with the curriculum, there will be a variety of tools in place.

**Curricular Tools**

1. **The Math Opportunities Chart**
   Co-developed by teachers in the summer workshop, this ensures that all required math concepts are being covered. It also scaffolds teachers in learning how to capitalize on these math opportunities. While this chart explicitly states when math concepts will be covered, as teachers become more comfortable with the material, they will become more proficient in identifying additional activities that can incorporate these math concepts.

2. **Math Problem reference tool**
   Developed by the teacher during the summer workshop, this tool provides examples of applicable math problems, grounded in the required mathematical concepts. The math problems are of the same mathematical concepts that the teacher intends to cover, but are specifically set in areas outside of the Antarctica program’s area.

3. **Assessment Rubric**
   Created by the teacher during the summer workshop, this tool allows teachers to evaluate students, as groups and as individuals, on how well they understand the processes and decisions involved in solving math problems related to the required mathematical concepts. It will be based on criteria determined by the teachers. Some “best practice” rubrics (as determined by the researchers) will be provided as examples. (See Appendix A for an example of a “Best Practice” rubric, which evaluates students based on how well they understood the problem, how they solved the problem, their decision process, and their outcome.)

4. **Online discussion forum and contact system**
   This system, provided with the curriculum, provides instructors with a venue for ongoing discussion and support. For novice instructors, the online discussion is allows them to learn from more experienced teachers. The contact system, which participating teachers are responsible for keeping updated with their current information, gives participating teachers a support mechanism for when challenges arise. Initially, this will also contain contact information for the project researchers and developers. As teachers become more experienced, it will become a self-sustaining teacher network.
Curricular Structure

Project Set-up

1. At the start of the program, the teacher will arrange the MMAP Activities into “Activity Blocks.” These will consist of three related activities, as appropriate, and will be explained to the students at a high level.

2. Prior to each Activity Block, the teacher will explain to the students in detail what activities they are going to walk through, as well as some examples of the math concepts they will be learning. Students will write a project overview for their parents, which the teacher will review. Due to the original curricular design, parents were concerned about their children’s learning. The intent here is to ensure that parents understand what the project will entail up front.

3. Though the sequence of activities is predetermined based on real-world logic, teachers will have worked together to determine which math concepts they will cover within each activity. Throughout the program, the teacher will refer to the Math Teaching Opportunities Chart – which teachers co-developed during their summer workshops – to ensure that the required mathematical concepts are being covered.

Student Involvement

4. Teams of three students (“triads”), assigned by the instructor to combine higher-level with lower-level students, will work through activities together sharing one computer. As the triads work, the teacher will rotate around the class, providing support as needed. (See Appendix B for seating suggestion)

5. For each activity, there will be a “Leader Team,” which will lead the discussion around that team’s activity. At the start of the program, students will determine which triad will be the Leader Team for which activity.

Checkpoints

6. After the completion of each Activity Block, the teacher will gather the students for a class discussion called a “checkpoint”. The Leader Team will be responsible for sharing its work processes and learnings with the class during this discussion. This is an opportunity for the teacher to assess how well the leader team understands the mathematical concepts.

7. After the Leader Team has shared its processes and learnings, students will assemble in larger teams, which will consist of one-third of the class. All students will be assigned to Team 1, Team 2 or Team 3, so that they will not be working with anyone from their triad. (See Appendix B for seating suggestion)

8. Using the Math Problem Reference tool, the teacher will give the larger teams math problems related to the concepts covered in the activities. Students will work with their larger teams in solving these problems, ensuring the knowledge is being shared
across groups.

9. The teacher will then ask the larger teams to share their solutions with the class as a whole. This will be disguised as low-stakes competition, with some type of reward (as the teacher deems appropriate) for active participation. As the teams explain how they came to their solution, the teacher will be able to assess how well the groups have learned the applicable mathematical concepts. The teacher will use the Assessment Rubric in evaluating how well the students are not only solving the problems, but also understanding their processes.

**Final Assessment**

10. After the culmination of the MMAP project, the teacher will perform a final assessment, using the Assessment Rubric, in order to determine how well the students have learned and understand the required math concepts. Over three days, students will work for part of the class, both in their groups and individually, in solving math problems relating to the MMAP project, but in other subject areas (e.g., rather than focusing on architecture, teachers may choose to base the problems on landscaping or travel). This allows the teacher to assess students as they participate in different work environments.

- **Small Teams**: For the first day, students will work in their triads.
- **Large Teams**: For the second day, students will work in their teams of one-third of the class.
- **Individual**: For the third day, students will work individually to solve similar problems.

11. Students will also write up a project summary, to be used as another method of assessment. For this assignment, students will be asked to choose three of the activities, and explain how their group came to their solution. They will be asked to include the mathematical equations which were used in their work processes. Also, they will be asked if they feel this was the best method, or if they thought there was a better approach, and to explain that as well.

**VII. References**

Appendix A

Assessment Rubric

This example of a “Best Practice” rubric evaluates students based on how well they understood the problem, how they solved the problem, their decision process, and their outcome.

Example Assessment Rubric

<table>
<thead>
<tr>
<th>PERFORMANCE LEVELS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
</tr>
<tr>
<td>Identified special factors that influenced the approach before starting the problem</td>
</tr>
<tr>
<td>Approach was efficient or sophisticated</td>
</tr>
<tr>
<td>Clearly explained the reasons for the correct decisions made throughout the problem</td>
</tr>
<tr>
<td>Solved the problem and made general rule about the solution or extended the solution to a more complicated situation</td>
</tr>
</tbody>
</table>

Source: Vermont Department of Education: *Vermont Math Problem Solving Criteria*. 
Appendix B

Suggested student seating arrangements

For “triad” teams working through MMAP activities:

For larger teams working through math problems during Checkpoints: