Long-Term Laddermill Modeling for Site Selection

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Non-powered flight vehicles such as kites can provide a means of transmitting wind energy from higher altitudes to the ground via tethers. At Delft University of Technology, construction and testing of such a high altitude wind machine is ongoing. The concept is called the Laddermill. It generates energy by pulling a line under high tension from a drum with a kite and retrieving it under low tension. The change in tension is achieved by changing the angle of attack and flight pattern of the kite. This paper presents a modeling and optimization approach that can be used to help design Laddermill systems for particular sites around the world. Some crude assumptions are used to derive the average power that can be produced by a Laddermill system, taking into the most important system parameters. Historical wind data is used to size the kite and cable diameter needed to produce a 1 MW machine for the least cost.

I. Introduction

\textsc{G}lobal wind energy has been growing at an average rate of 30\% during the last 10 years. The year 2006 saw the installation of 15,197 MW, taking the total installed wind energy capacity to 74,223 MW, up from 59,091 MW in 2005 [1]. Europe is the world market leader, having 65\% or 48,062 MW of the global market. Estimated growth rates made by the GWEC predict that by the year 2010 the global wind market will have doubled compared to the situation in 2006 [1]. In terms of economic value, the wind energy sector has now become firmly established as one of the important players in the energy markets, with the total value of new generating equipment installed in 2006 reaching €18 billion [1].

At higher altitudes, more energy is contained in the wind than at lower altitudes, as can be seen in Fig. 1, which shows the average wind profile at a particular site in The Netherlands. There is a clear trend towards larger Horizontal Axis Wind Turbines (HAWTs) [1]. The growth of HAWTs however is limited by the cost of putting the system at the desired altitude, and by the increasing rotor diameter required, which results in a very low rotational speed. Due to these deficiencies, a novel concept for harnessing the tremendous source of energy at higher altitudes has been developed called the Laddermill. This concept is described in the following Section.

![Fig. 1. Wind speed at higher altitudes [2].](image-url)
II. The Laddermill

An alternative method of extracting energy from the wind is the Laddermill [3]. The Laddermill generates energy from the wind by pulling a cable from a drum using kites, as shown in Fig. 2. While the cable is pulled off the drum, the kites are controlled such that they deliver a high pulling force to the cable. When there is no cable left on the drum, the kite is pulled back in. During the reel-in maneuver, the kites are controlled so that there is minimum tension in the line. The difference in tension allows net power to be generated per cycle. When the portion of the cable that was pulled out is fully retrieved, the cycle is repeated. The Laddermill concept was successfully tested in 2007 [4].

To achieve a high tension in the power generation phase, the kite is flown across the wind at high speed, in a pattern that may look like the ones shown in Fig. 3. The speed at which the kite can fly across the wind is determined by the lift-to-drag ratio of the kite and cable system as explained in [7]. In (the solid part of) the trajectory of Fig. 3 the kite is moving 5.8 times faster than the wind speed, or at 69 m/s [5], increasing the tension in the line by a factor of $(5.8)^2$ or 33. This is one of the key advantages of using kites to produce power: by flying across the wind, power can be extracted more cheaply than by using a conventional wind turbines.

In the study of Houska [5], the angle of attack of the kite is reduced during the retraction phase. While the L/D (of the model used) increases, the L/D of the whole system decreases because the cable drag coefficient remains the same. Current tests of this technology have been performed with commercially available surf kites. Surf kites are used because they have reasonable L/D, are readily available and can deliver the right pull for a 1-10 kW system.
Fig. 3. Optimal path for kite power extraction a) Houska et al. [5], b) Williams et al. [6]

Automatic control of the kite is currently under investigation, and a proprietary control algorithm has been developed that can steer a kite along a desired trajectory and is robust to large disturbances [7]. To succeed in automatic control for optimal power generation, the kite needs to be modeled, and optimal paths for power extraction need to be determined in real-time or close to real-time. For determination of optimal paths, the kite is modeled as a point mass [5]-[6], but more advanced kite modeling is also performed, where the steering mechanism and the flexibility of the kite are taken into account as shown in Fig. 4.

Fig. 4. Model of a flexible kite with adjustable tow point positions [8].
By fitting the kite models to actual measurement data, the properties of the kite can be determined and the kite models can be improved. When the model is sufficiently accurate, state estimation can be performed to enable feedback control even if not all states can be measured accurately [9]. To fly the geometric patterns as shown in Fig. 3, it is essential that good control is achieved over the kite. The kite and the steering mechanism that are used currently are shown in Fig. 5.

![Kite and steering mechanism](image1)

**Fig. 5. Steering mechanism and kite.**

It is expected that a Laddermill can generate energy more cost efficiently than HAWTs. Several reasons for this are given:

A. **Wing efficiency**

The wingtips are responsible for most of the energy generated in the HAWT, because they travel at the highest velocity, as indicated in Fig. 6. The inner part of the blades travels at lower speeds and contributes less to the total power. As shown in Fig. 6, the wing of the Laddermill can fly exactly the path of the wingtip of a HAWT blade, such that the whole wing is used efficiently.

![Wing efficiency comparison](image2)

**Fig. 6. Wing efficiency of a conventional wind turbine compared with a kite.**

B. **Wing cost**

The blades of a HAWT are complicated, double curved objects, as can be seen in Fig. 7. Estimations show that the lifting body of a Laddermill will be about equal in size to the blades of a HAWT for equal installed power. However, the L/D requirements for the Laddermill wing are quite mild, which makes production of the wing relatively straightforward. An L/D of more than ~15 only results in a limited increase in performance because cable drag becomes dominant. An approximation of the Laddermill wing area for several installed powers in shown in Fig. 8, for a wind speed of 12 m/s. This paper will develop better measures of Laddermill performance.
C. Wing loading

The blades of a windmill are loaded quite unfavorably. Because of their length, bending moments are quite large. Also, the rotation causes significant centrifugal forces. In the Laddermill the bending moments can be kept low by means of bridling, dependent on the number of bridles.

D. Gearbox & generator

The rotation speed of a HAWT is limited by the blade tip speed. For large wind turbines this results in slow rotation. A 5 MW HAWT revolves at 12.1 RPM at the maximum power. The rotation speed of the Laddermill is limited by the drum diameter, which should be about 20x larger than the cable diameter. For a 5 MW Laddermill, at maximum power the cable moves at about 4 m/s, resulting in about 1 Hz or 60 rpm. This means that the forces on the gearing are 5 times smaller than for the HAWT, resulting in a much smaller and cheaper gearbox.
E. **Ground station**

The ground station of the Laddermill, where the generator and the gearbox are located, must cope with the forces exerted by the cable. These forces are smaller than the forces exerted by the blades of a HAWT, because the rotational speed of the Laddermill is smaller. This results in a more lightweight and cheaper structure on the ground.

F. **Tower & Foundation**

The 120 m high tower of a large windmill like the REpower 5M constitutes a substantial part of the total cost, about 20%, or 1 M€. The 1200 m long cable for a 5MW Laddermill, which takes the Laddermill to 500m altitude costs about 50 k€.

G. **Capacity factor**

The capacity factor is defined by the average power generated over a year, divided by the installed power. The capacity factor of all HAWTs world wide was 22.2% in 2002. Simulations of a Laddermill in the historical wind data from the KNMI show that a Laddermill should have a capacity factor of over 50%. On the other hand it should be noted that several issues need to be resolved before Laddermills can become commercially interesting.

In the following sections, numerical optimization is used to design the best Laddermill system configurations at any selected site using historical wind data.

III. **Approximation of Laddermill Power Production**

The Laddermill system is analyzed by considering a single kite located at the desired altitude \( h \). We assume for the purposes of this analysis that the kite is moving with a constant velocity given by \( v_k \). The wind speed at the altitude \( h \) is given by \( w \). The effective wind velocity at the kite is given by

\[
\mathbf{w}_e = \mathbf{w} - v_k
\]  

(1)

as shown in Fig. 9.

![Fig. 9. Basic aerodynamic forces on the kite.](image)

As shown in [5], the effect of kite mass has a relatively small effect on the overall power production of a kite generating system and hence it is neglected in this analysis. The lift of the kite can be approximated as
\[ F_L = \frac{1}{2} C_L S \rho \| w_e \| w_e \times k \]  

(2)

where \( k \) is a vector out of the page, \( \rho \) is the density of the air, \( C_L \) is the kite lift coefficient, \( S \) is the kite area. The kite drag is similarly given by

\[ F_D = \frac{1}{2} C_D S \rho \| w_e \| w_e \]  

(3)

where \( C_D \) is the kite drag coefficient. The influence of cable drag on the system can be approximated by assuming that the effective wind at any point on the cable is a linear function of the effective velocity at the kite \([5]\)

\[ w_e(s) \approx \frac{s}{L} w_e \]  

(4)

where \( L \) is the cable length, and \( s \in [0, L] \) is a spatial coordinate along the cable. The drag force on the cable is the integral of the contributions of each element \( ds \) of the cable such that total moment due to drag is given by

\[ M_c = \int_0^L \frac{1}{2} C_{D_c} \rho d \frac{s^3}{L^2} \| w_e \| w_e \| s \| ds = \frac{1}{8} C_{D_c} \rho d L^2 \| w_e \| w_e \]  

(5)

where \( C_{D_c} \) is the cable drag coefficient, \( d \) is the cable diameter. Hence, the equivalent force at the kite is given by

\[ F_c = M_c / L = \frac{1}{2} \frac{1}{4} C_{D_c} \rho d L \| w_e \| w_e \]  

(6)

The lift to drag (glide) ratio of the entire system is therefore given by

\[ G = \frac{F_L}{F_D} = \frac{C_L}{C_D + \frac{d L C_{D_c}}{4 S}} \]  

(7)

If the kite moves at constant speed, we can obtain the kite speed as a function of wind speed and the glide ratio

\[ \| v_k \| = G \| w \| \]  

(8)

If the cable speed is constant, then the cable tension is calculated to be
Note that the angle of the cable to vertical \( \theta \) has not yet been included in the analysis. It is straightforward to rotate the wind component in the kite frame, under the assumption that the wind remains in the horizontal plane.

The power produced during the power generation phase is given by

\[
P = T \dot{L}
\]  

(10)

The power consumed during the reel-in phase is given by

\[
P_r = T_r \dot{L}_r
\]  

(11)

where \( T_r \) is the tension during the reel-in phase. The total cycle time is made up of the power generation and retrieval phases, which is approximated as

\[
t_c = \frac{\Delta L}{L} + \frac{\Delta L}{L_r}
\]  

(12)

where \( \Delta L \) is the amount of cable that is reeled-in and out per cycle. Assuming a mechanical efficiency of \( \eta \), the total energy generated per cycle is given by

\[
E = \eta P \frac{\Delta L}{L} - \frac{P_r}{\eta} \frac{\Delta L}{L_r}
\]  

(13)

Hence, the average useful power generated per cycle is calculated as

\[
P_{useful} = \frac{E}{t_c}
\]  

(14)

Finally, the maximum allowed force in the cable is given by

\[
F_{\text{max}} = \frac{\sigma_{ut}}{F_s} \pi d^2 / 4
\]  

(15)

where \( \sigma_{ut} \) is the material ultimate strength, and \( F_s \) is the factor of safety. We utilize the above equations to optimize the kite size and cable requirements based on wind data provided as a function of altitude.

IV. Site Analysis Optimization Problem

To design an optimal system, we must first define what we consider an optimal system. We assume that the cost of system is driven as a function of the cable tension and kite size. If we use a glider plane as
opposed to a fabric kite, then the cost of the main spar is driven by the design load, which is directly related to the cable tension. The size of the kite influences the cost of construction, maintenance, launching costs, and kite mass. Therefore, it can be argued that keeping the kite size as small as possible is beneficial – it also minimizes safety issues when something goes wrong. Hence, we define an optimal system as one that maximizes the average power per unit cost. Conversely, the seek to minimize the following cost

$$J = -\frac{\bar{P}}{\bar{T} + k_1 S}$$

where \(\bar{P} = \frac{1}{n} \sum_{i=1}^{n} P_i\) is the average power generated over \(n\) equally spaced intervals (days), \(\bar{T} = \frac{1}{n} \sum_{i=1}^{n} T_i\) is the average tension, \(S\) is the kite area, and \(k_1\) is a weighting coefficient.

In minimizing Eq. (16) it is important to take into consideration several constraints. The first is that the cable diameter must be sufficient to sustain the loads it is subjected to. Hence, for each day of data we enforce the constraint

$$T_i \leq F_{\text{max}}, \quad i = 1, \ldots, n$$

In the optimization problem we allow the cable length to be optimized. However, the cable cannot be too short. We fix the amount of cable that is reeled \(\Delta L\) and constrain the total length ratio to be above a threshold

$$L / \Delta L \geq \gamma$$

Finally, we seek to design a particular class of system. This means we constrain the average power generated over the time of analysis to be equal to some value, 1 MW for example,

$$\bar{P} = P_d$$

The optimization parameters are the following: kite altitude \(h\), kite angle to the vertical \(\theta\), cable diameter \(d\), kite area \(S\), and kite lift coefficient on each day \(C_{L_i}\), \(i = 1, \ldots, n\). We fix the reel out speed of the cable to be 1/3 of the wind speed.

V. Results

The nominal case we have selected uses wind data from [14] at the location de Bilt in the Netherlands from the period 1998 to 2007. The data is provided at discrete altitudes that are not constant from day to day. Therefore, the wind data is linearly interpolated as a function of altitude to obtain the wind speed at any desired altitude for the day under consideration. We have fixed the following system parameters: \(\rho = 1.225 \text{ kg/m}^3\), \(F_s = 2.5\), \(\eta = 0.92\), \(C_{D_\infty} = 1.2\), \(\sigma_{ut} = 1.6 \text{ GPa}\), \(\Delta L = 200 \text{ m}\), \(\dot{L}_i = 16.7 \text{ m/s}\). We have assumed that the tension during the reel-in phase is 1/50 of the tension during power generation. The maximum lift coefficient of the kite is 1.2. In this initial optimization, \(G = 16\), and 3 years of wind data is used. We constrain the minimum angle of the cable to the vertical to be 45 deg with a maximum of 80
deg. The maximum percentage of cable that can be reeled is 40%. This combined with ΔL = 200 m gives a minimum cable length of 500 m.

The results of the optimization for the case of a 1 MW machine with 3 years worth of wind data are as follows:

<table>
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<tr>
<th>Parameter</th>
<th>Optimal Value</th>
<th>Units</th>
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<tbody>
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<td>Kite Altitude</td>
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<td>m</td>
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<tr>
<td>Cable Angle</td>
<td>45</td>
<td>deg</td>
</tr>
<tr>
<td>Cable Diameter</td>
<td>29.68</td>
<td>mm</td>
</tr>
<tr>
<td>Kite Area</td>
<td>312.4</td>
<td>m²</td>
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<tr>
<td>Average Kite $C_L$</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 10 shows an effective time series of the power generated by the Laddermill. The results show that the majority of the time the system operates at approximately 1 MW output. However, the peak outputs from the system can be at up to 2.4 MW. Fig. 11 summarizes the power output using a histogram. In normal operation, the power is nearly normally distributed around the desired output. However, there are a number of occasions where the system operates at very small output levels. These correspond to times when the wind speed drops significantly from the norm at the selected optimal altitude, or when the system enters into a low power producing mode (this mode is a consequence of constraining the average power output to be 1 MW; in reality, the system would always operate at its optimal output resulting in an average power output greater than 1 MW). Based on this, it would be preferable to constrain the average output using an inequality inside of an equality, i.e., the average power should be at least 1 MW. Fig. 12 shows a histogram of the lift coefficient, which shows that the kite rarely needs to operate at its maximum lift coefficient for this configuration. Finally, Fig. 13 shows a histogram of the cable tension. This illustrates that the system is optimal for the wind conditions, i.e., over 80% of the time the system operates close to the peak tension load. This means that the peak power outputs are due to higher wind speeds, resulting in faster reeling out, rather than peaks in the tension.

Fig. 10. Power generated for historical wind data.
Fig. 11. Histogram of average power generated over each data index.

Fig. 12. Histogram of kite lift coefficient.
A. Sensitivity Analysis

It is very useful to quantify the effects of several of the assumptions used to derive the optimal system shown above. In this section, a sensitivity analysis is performed by resolving the optimization problem with different input parameters. However, in order to reduce computation time, only a subset of the original data is used.

1. Effect of Altitude and Cable Angle

Since the effect of changing the kite altitude and cable angle combine to determine the cable length variations in them should be considered together. Fig. 14 summarizes the results as functions of the two variables. The results show that the power per unit cost scales almost linearly with changes in angle and altitude. It shows that for a fixed cable angle, increasing the kite altitude leads to a decrease in performance. This is due to the increase in cable length and therefore cable drag. Similarly, fixing the kite altitude and increasing the cable angle results in an increase in cable length and decrease in performance. However, the two effects are not identical, since the wind speed is higher at higher altitudes. The required cable diameter decreases as the kite altitude increases, but only marginally. The cable diameter is virtually insensitive to changes in cable angle, indicating that it is the influence of the change in wind speed that is affecting the cable diameter. The required kite area varies nonlinearly with respect to altitude and cable angle. The results show that smaller kites are required at higher altitudes. Furthermore, as the cable becomes more aligned with the vertical the cable drag increases, requiring larger kites to compensate. However, there is a balance between the two and a minimum can be seen in the surface in Fig. 14. This shows an optimal kite area in the region of 60 deg. This angle is roughly the optimal angle produced through dynamic optimization of kite control for maximum power generation in a fixed wind condition.
2. Effect of Glide Ratio

The glide ratio of the system is a key parameter governing the cable force. However, Fig. 15 shows that decreasing the glide ratio by nearly 69% results in a drop in performance of only approximately 3%. To compensate however, much larger kites are needed. The average power per unit cost would be more strongly affected if the kite area is penalized more in the cost function. The required kite area increases by nearly a factor of 5 for a factor of 3 drop in glide ratio.

3. Effect of Reel-In Speed

The retrieval speed of the cable is another important parameter that affects the performance. This is because shorter retrieval times increase the rate of energy that can be generated and therefore the power output. It is not surprising to see a rapid (cubic) decline in performance with decreases in the reel-in
speed. To make up for the power difference, the kite area must be increased, together with an increase in the cable diameter.

![Graph showing effect of reel-in speed on system performance.](image)

Fig. 16. Effect of reel-in speed on system performance.

4. **Effect of Reel Length**

The amount of cable allocated for reeling in and out affects the cycling times, as well as (indirectly through constraints), the total cable length. Fig. 17 shows that the system performance is increased if smaller cable amounts are used for reeling. The kite altitude increases as a linear function of the reel length with the cable angle fixed until about $\Delta L = 450$ m. At this point the cable angle increases and the altitude remains constant. Fig. 17 shows that decreasing the reel length means that larger kites are required with larger diameter cables. However, $\Delta L = 200$ m seems like a reasonable compromise, which is the value used to design the baseline system shown above.

![Graph showing effect of cable length used for reeling on system performance.](image)

Fig. 17. Effect of cable length used for reeling on system performance.
5. Effect of Retrieval Tension

The retrieval tension is an indicator of the overall efficiency of the energy conversion process. If the tension cannot be decreased to some reasonable percentage of the tension used in the power generation phase, then net power will not be generated. This is because reeling the cable out decreases the apparent wind at the kite, whereas reeling the cable in increases the apparent wind. In the model used in this paper, the retrieval tension is set to be a fixed fraction of the tension in the power generation phase. Hence, increasing this fraction leads to a linear decrease in performance. The cable diameter and kite area needed to compensate change by approximately 5% and 10%, respectively, for a 10% change in the retrieval tension.

Based on the above findings, the system was reoptimized with a slightly different cost and constraints. The cost function was modified to be

\[ J = -\frac{P}{T S d m_c} \]

where \( m_c \) is the cable mass. This removes the influence of different weightings on the terms and measures cost as being proportional to mean tension, kite area, cable diameter, and cable mass. The effect of power electronics has not yet been included, and this is relegated to future work. We constrain the cable angle to be at least 57 deg based on Fig. 14. Furthermore, we do not constrain the mean power output to be exactly 1 MW. Instead, we constrain the mean power to be at least 1 MW - so it can be more than that if it is optimal. Finally, we model the kite drag as a function of the lift coefficient in Eq. (7) as \( C_D = 0.03 + 0.04C_L^2 \) thus accounting for the fact that the optimal L/D cannot always be flown. The results for the optimal design are summarized in Table 2 using 1 year of wind data.

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<th>Parameter</th>
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<th>Units</th>
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<td>Cable Angle</td>
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<tr>
<td>Cable Diameter</td>
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<tr>
<td>Kite Area</td>
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<tr>
<td>Average Kite ( C_L )</td>
<td>1.01</td>
<td></td>
</tr>
</tbody>
</table>

The results in Table 2 show that the kite area can be significantly reduced by flying at a lower altitude. This is at the expense of a slightly thicker cable. This is reasonable, since the cable drag due to the wind decreases as the angle to the vertical decreases. Fig. 19 summarizes the power output and lift coefficient of the kite after running the “design” through 10 years worth of wind data. On each day, the lift
coefficient and reeling speed as a percentage of the wind speed is re-optimized to give the maximum power output, subject to the constraint on allowable tension. Fig. 19 shows the case where the upper bound on reeling speed is 1/3 of the wind speed. Here we see that the system is underperforming, although the power output is “close” to 1 MW. Better results can be obtained by increasing the maximum allowable reeling speed to ½ of the wind speed. The results for the second case are shown in Fig. 20. The average power output meets the design requirement of 1 MW, with peak power outputs up to 3.5 MW.

Fig. 19. Histograms of power output and lift coefficient for 10 years of wind data with maximum reeling speed constrained to be 1/3 of the wind speed.

Fig. 20. Histograms of power output and lift coefficient for 10 years of wind data with maximum reeling speed constrained to be 1/2 of the wind speed.
VI. Conclusions

An optimization procedure has been developed for sizing Laddermill-type systems using long-term historical wind data. The result of the optimization gives the kite size, altitude, cable length and cable diameter that maximizes the power output per unit cost.

References